

AC BRIDGES

1. Preparation questions (20 points)

Answer the following questions before the experiment and present them in a report before the experiment.

1. What does ohmic, capacitive and inductive load mean? Please explain briefly.
2. How does the impedance of an ohmic load change with frequency? Please explain briefly.
3. How does a capacitive load change with frequency? Please explain briefly.
4. How does an inductive load change with frequency? Please explain briefly.
5. How are the capacitance and inductance parameters measured today? Please explain briefly.

2. Finding Coil Parameters

The low frequency equivalent of a coil without iron is shown in Figure 1.

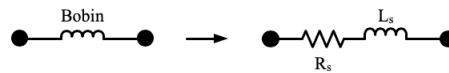


Figure 1 Low frequency equivalent of the coil

Here (Figure 1), R_s shows the wire resistance and L_s shows the inductance of the coil. As the capacitances between the windings are negligible as long as they are operated at low frequencies, they are not shown as equivalent.

Let's now find the coil parameters using a suitable alternative bridge (Figure 2). As it is known, in order to balance the alternating current bridges, it is necessary to provide (1-3) correlations between the impedances in their arms.

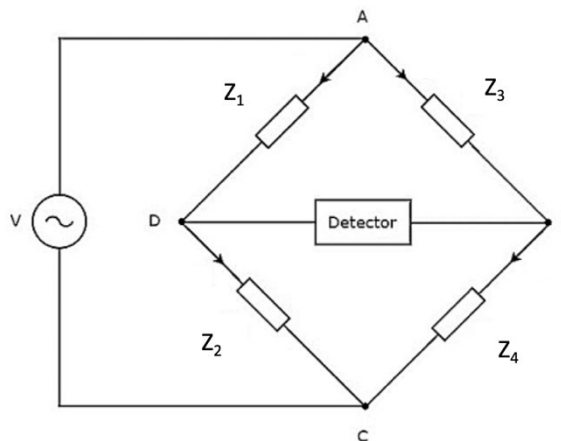


Figure 2 Theoretical AC bridge circuit

$$Z_1 \cdot Z_4 = Z_2 Z_3 \quad 1$$

$$|Z_1|e^{j\theta_1} \cdot |Z_4|e^{j\theta_4} = |Z_2|e^{j\theta_2} \cdot |Z_3|e^{j\theta_3} \quad 2$$

$$|Z_1| \cdot |Z_4|e^{j(\theta_1+\theta_4)} = |Z_2| \cdot |Z_3|e^{j(\theta_2+\theta_3)} \quad 3$$

When equation (1) is satisfied, consequently (4, 5) are also satisfied.

$$|Z_1| \cdot |Z_4| = |Z_2| \cdot |Z_3| \quad (4)$$

$$\theta_1 + \theta_4 = \theta_2 + \theta_3 \quad (5)$$

In order to simplify the circuit, components are taken as purely ohmic thus $\theta_2 = \theta_3 = 0$.

So, phase relations are simplified to (6):

$$\theta_1 + \theta_4 = 0 \quad (6)$$

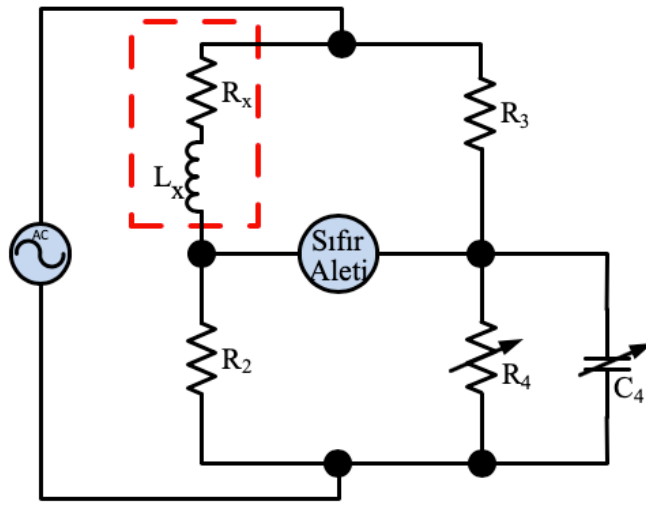


Figure 2. Maxwell-Wien bridge

In order for an AC bridge (Figure 3) in practice from this connection to balance, the phase angle of the impedance in the fourth arm must be equal to the negative of the phase angle of the unknown arm of the bridge. The impedance of the coil we want to measure at low frequencies is as follows:

$$Z_1 = R + j\omega L = \sqrt{R^2 + (\omega L)^2} \cdot e^{-j\arctan(\frac{\omega L}{R})} \quad (7)$$

The angle of the coil (θ_1) is positive. Therefore, the phase angle of the circuit that will stabilize the bridge must be negative. The simplest circuits with a negative phase angle are circuits consisting of a capacitor and a resistor.

Equations (8-9) are obtained if a capacitor is connected in series with a resistor, and equations (10-11) are obtained if connected in parallel.

$$Z_4 = R_4 + \frac{1}{j\omega C_4} = \sqrt{R_4^2 + \frac{1}{(\omega C_4)^2}} \cdot e^{-j\arctan(-\frac{1}{\omega C_4 R_4})} \quad (8)$$

$$\theta = -\arctan(\frac{1}{\omega C_4 R_4}) \quad (9)$$

$$Z_4 = \frac{1}{R_4} + j\omega C_4 = \frac{R_4}{\sqrt{1 + (\omega C_4 R_4)^2}} \cdot e^{-j\arctan(\omega C_4 R_4)} \quad (10)$$

$$\theta_4 = -\arctan(\omega C_4 R_4) \quad (11)$$

Equation (12) is obtained for the capacitor-resistance series circuit and (13) for its parallel circuit.

$$\arctan\left(\frac{\omega L}{R}\right) = \arctan\left(\frac{1}{\omega C_4 R_4}\right) \quad (12)$$

$$\arctan\left(\frac{\omega L}{R}\right) = \arctan(\omega C_4 R_4) \quad (13)$$

In the second of these relation is more useful because ω 's cancel each other out. Thus impedances on the arms of the bridge were chosen and $\frac{L_x}{R_x} = C_4 R_4$ equality is obtained.

Let's build the bridge. In this bridge, known as the Maxwell-Wien bridge, the coil parameters are found by comparing them with a capacitor and resistor. By choosing Z_2 and Z_3 as resistors, Z_4 impedance is provided to be capacitor and resistor. If Z_4 had been selected as resistance, the impedances of Z_2 and Z_3 would have to consist of one resistor and coil. In practice it is more difficult to build inductance circuit that just a ohmic circuit. Let's find the parameters of the coil by taking advantage of the equation (1), when the bridge in Figure 3 in equilibrium. The values of the components in the circuit are replaced by the equation numbered (14) and the coil parameters are found in the equations numbered (16-17).

$$Z_1 = Z_2 Z_3 \cdot \frac{1}{Z_4} \quad (14)$$

$$R_x + j\omega L_x = R_2 R_3 \left(\frac{1}{R_4} + j\omega C_4 \right) \quad (15)$$

When we make the real and imaginary parts equal within each other, we find the following equations

$$R_x = \frac{R_2 R_3}{R_4} \quad (16)$$

$$L_x = R_2 R_3 C_4 \quad (17)$$

2. 1. Running the experiment (40 points)

Please follow the instructions below:

1. Setup the circuit in Figure 3.
2. Take $V_p=10V$, 400 Hz for a sinusoidal voltage source. In Multisim, replace the null device with a cable and record current (I_{rms}) in this element.
3. Take R_2 as 300Ω , R_3 as 1100Ω , R_4 as 600Ω and C_4 as $1\mu F$, then find the corresponding R_x and L_x values and the resulting impedance (Z_x).
4. Use the values you obtained in the step 3 and by replacing the R_4 and C_4 fill the table below. Explain the curve for current vs impedance amplitude for $(R_4//C_4)$ coming out of the table (It will be done in Multisim).

$R_4 (\Omega)$	$C_4(\mu F)$	$I_{rms}(mA)$
100	0.2	
225	0.4	
350	0.6	
475	0.8	
600	1	
725	1.2	
850	1.4	
975	1.6	
1100	1.8	

3. Finding Capacitor Parameters

Use the parallel equivalent circuit, shown in Figure 4, to measure the parameters of the capacitor. Here, the capacitor impedance is revisited as in equation (18).

$$Z = \frac{1}{R} + j\omega C = \frac{R_4}{\sqrt{1 + (\omega C_4 R_4)^2}} \cdot e^{-j\arctan(\omega C_4 R_4)} \quad (18)$$

From the relations in (5-6), we take θ_x as the phase of the capacitor, we obtain the following relation; $\theta_x + \theta_4 = \theta_2 + \theta_3$. Since, 2nd & 4th components are ohmic, we get $\theta_2 = \theta_4 = 0$. Thus, for a balanced circuit θ_x should be equal to θ_3 .

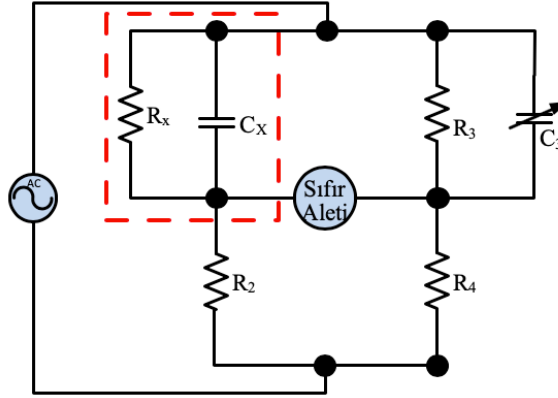


Figure 3. Finding capacitor parameters with a Wien bridge

This bridge is known as the Wien bridge. The expression of the bridge is in equilibrium condition (19). If the real and virtual parts of this expression are made equal to each other, the capacitor parameters (capacitance and capacitor internal resistance) become as in equations (19-24).

$$R_4 \cdot \frac{R_x}{1 + j\omega C_x R_x} = R_2 \cdot \frac{R_3}{1 + j\omega C_3 R_3} \quad (19)$$

$$R_4 R_x (1 + j\omega C_3 R_3) = R_2 R_3 (1 + j\omega C_x R_x) \quad (20)$$

$$R_4 R_x = R_2 R_3 \quad (21)$$

$$R_x = \frac{R_2 R_3}{R_4} \quad (22)$$

$$C_3 R_3 R_4 R_x = C_x R_2 R_3 R_x \quad (23)$$

$$C_x = C_3 \frac{R_4}{R_2} \quad (24)$$

3.1. Running the experiment (40 points)

Please follow the instructions below:

1. Build the Wien bridge.
2. Take $V_p=10V$, 1000 Hz for a sinusoidal voltage source.
3. Take R_2 as $1k\Omega$, R_3 as 20Ω , R_4 as 330Ω ve C_3 as $470\mu F$, then find R_x and C_x and impedance value (Z_x).
4. Use the values you obtained in the step 3 and by changing C_3 with the given values in the table below, find the I_{rms} values. Explain the current vs capacitance curve coming out of the table (It will be done in Multisim).

$C_3(\mu F)$	$I_{rms}(mA)$
190	
260	
330	
400	
470	
540	
610	
680	
750	
820	